AP Calculus
CHAPTER 5 WORKSHEET
INTEGRALS

Name____________________________________

Seat # ________ Date _____________________

Approximating area using Riemann sums

1. a) Approximate the area under the graph of $f(x)=\frac{1}{x}$ from $x=1$ to $x=5$ using the **right endpoints** of four subintervals of equal length. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
   b) Repeat part a) using **left endpoints**.

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2. Approximate the area under the graph of $f(x)=25-x^2$ from $x=0$ to $x=5$ using the **midpoints** of five subintervals of equal length. Sketch the graph and the rectangles.

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3. a) Approximate the area under the graph of $f(x)=x^2+1$ from $x=-1$ to $x=2$ using the **right endpoints** of three subintervals of equal length. Sketch the graph and the rectangles.
   b) Improve your estimate by using six subintervals.
   c) Repeat parts a) and b) using **left endpoints**.
   d) Repeat parts a) and b) using **midpoints**.
   e) From your sketches in parts a), c), and d), which appears to be the best estimate?

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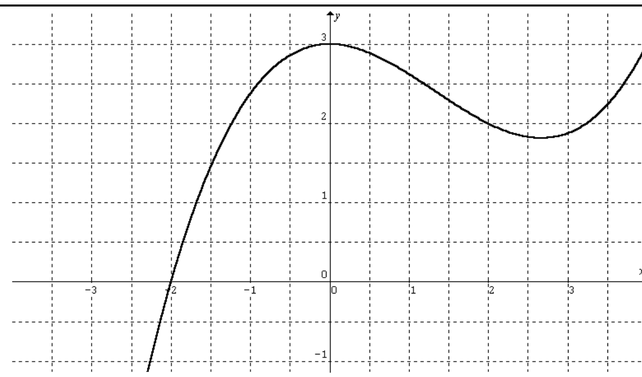
4. a) Approximate the area under the graph of the function shown to the right from $x=-2$ to $x=3$ using the **right endpoints** of five subintervals of equal length.
   b) Repeat part a) using **left endpoints**.
   c) Repeat part a) using **midpoints**.



5.

x	–5	–3	0	1	5
$f(x)$	10	7	5	8	11

   a) Given the values for $f(x)$ on the table above, approximate the area under the graph of $f(x)$ from $x=-5$ to $x=5$ using the **left endpoints** of four subintervals.
   b) Repeat part a) using **right endpoints**.
   c) Could you do this problem using **midpoints** of four subintervals? Explain.

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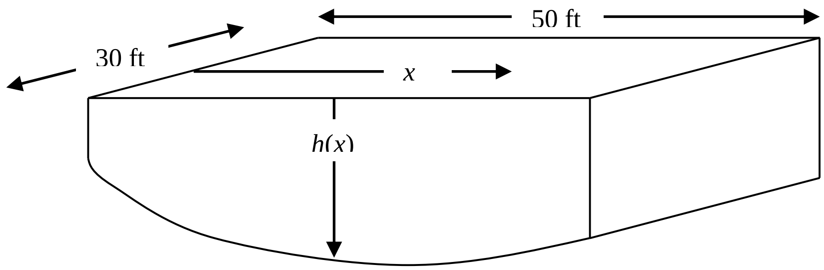
6. Left, midpoint, and right Riemann sums were used to estimate the area between the graph of $f(x)$ and the x-axis on the interval [3, 7]. We know that f is a function such that $f(x)>0$ and $f'(x)<0$ on [3, 7]. The same number of subintervals were used to produce each approximation. The estimates were 1.345, 1.578, and 1.723.

   Which rule produced each estimate? Justify your answer.

SEE OTHER SIDE

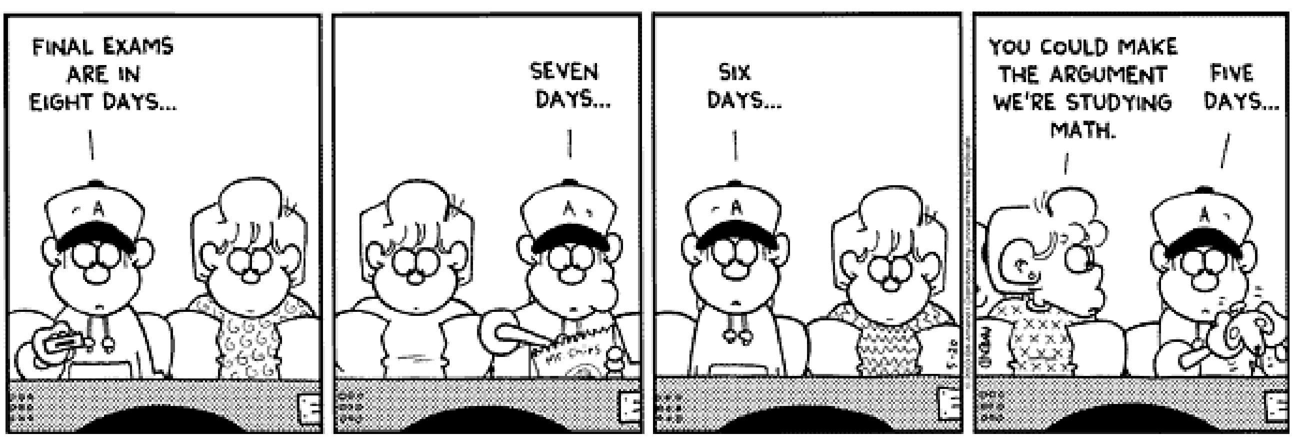
7. A swimming pool with a rectangular surface is 30 ft wide and 50 ft long. The volume of the pool is shaped as a prism (see drawing.) The table below shows the depth $h(x)$ of the water at 5-ft intervals from one end of the pool to the other.

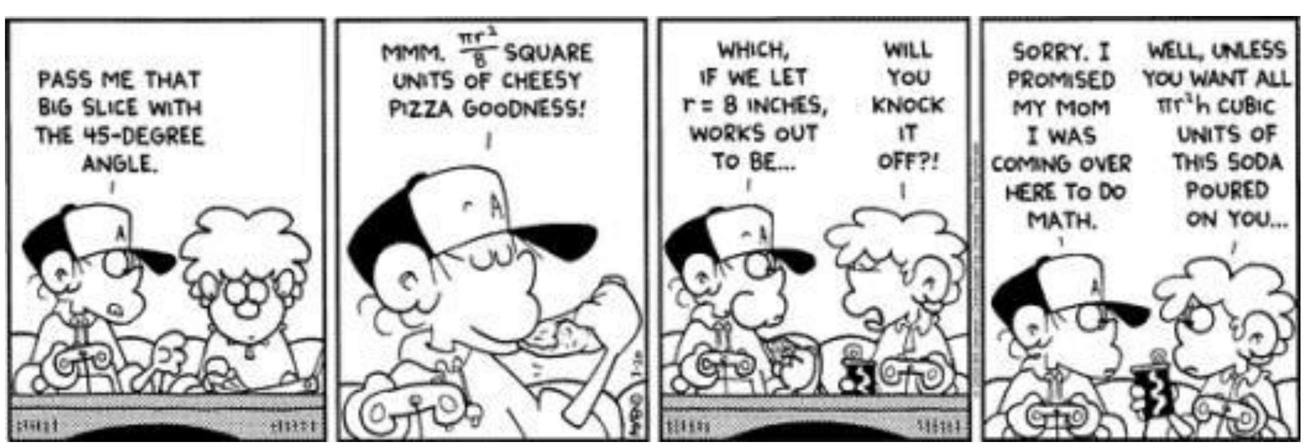


a) Estimate the lateral area of the pool using a Riemann sum with the **midpoints** of five subintervals of equal length.

b) Use this information to calculate the volume of water in the pool. *(Hint: remember that the volume of a prism is the area of its base times the distance between the bases.)*

Data for the swimming pool											
Position (ft): x	0	5	10	15	20	25	30	35	40	45	50
Depth (ft): $h(x)$	6.0	8.2	9.1	9.9	10.5	11.0	11.5	11.9	12.3	12.7	13.0





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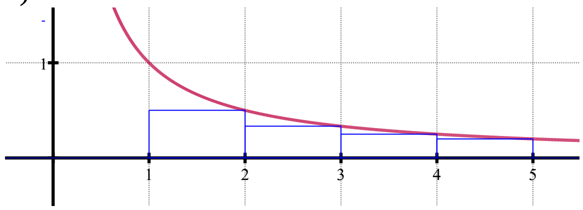
ANSWER KEY

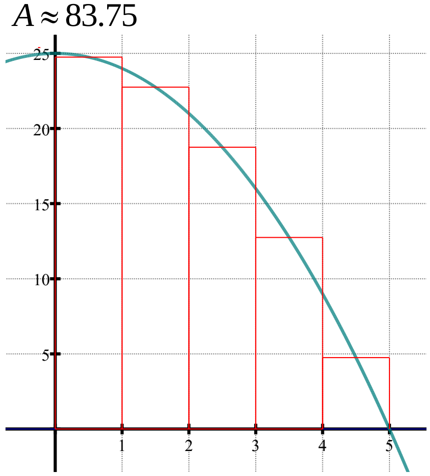
Approximating area using Riemann sums

1. a) $A \approx 1.283$. Underestimate.

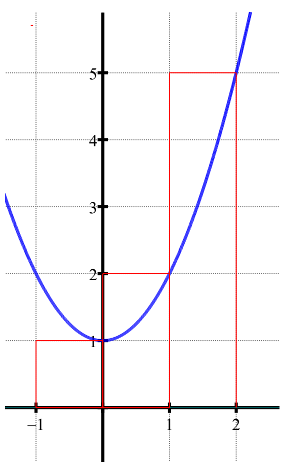


b) $A \approx 2.083$. Overestimate.

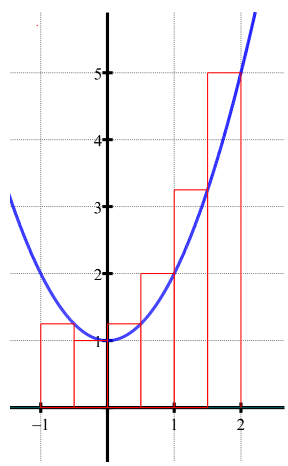
2. $A \approx 83.75$



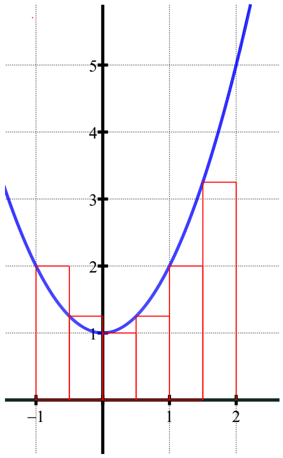
3.



a) $A \approx 8$



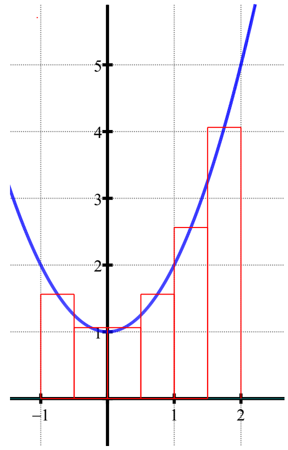
b) $A \approx 6.875$



3 subintervals:
$A \approx 5$

6 subintervals:
$A \approx 5.375$

c)



3 subintervals:
$A \approx 5.75$

6 subintervals:
$A \approx 5.938$

d)

e) Midpoint method with 6 subintervals

4. a) $A \approx 11.7$
   b) $A \approx 10$
   c) $A \approx 11.1$

5. a) Left endpoints: $A \approx 78$
   b) Right endpoints: $A \approx 81$
   c) No. We do not know the values for $f(x)$ at the midpoints of each interval.

6. Since $f'(x) < 0$ on [3, 7], the function $f(x)$ decreases on [3, 7]. This means that in each subinterval the values of the function for the left endpoint are higher than both the values for the right point and the values for the midpoint of the subinterval. It also means that the values of the function for the right endpoint of each subinterval are the lowest for that subinterval. So the estimates were 1.345 for the right Riemann sum, 1.578 for the midpoint sum, and 1.723 for the left Riemann sum.

7. a) Lateral area $\approx$ 537 sq ft
   b) Volume = Lateral Area $\times$ 30 $\approx$ 16110 cu. ft